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Preface

This guide accompanies *Calculus: Early Transcendentals* by Briggs, Cochran, Gillett, and Schulz. Think of it as a roadmap to the textbook and a collection of resources for use in your course. Though one might identify the main audience of this book as graduate students or instructors early in their careers, our hope is that seasoned professors will also benefit from the material offered here.

Bernard Gillett and Anthony Tongen

Features of the Text

The essential features of *Calculus: Early Transcendentals* are spelled out in its preface. We encourage you to become familiar with all aspects of the text, including its online components, so that you can make informed decisions about what to incorporate into your courses. The most important features are highlighted here.

- Make students aware of the explicit connection between the worked examples in the text and the *Basic Skills* exercises. Each block of *Basic Skills* exercises is directly linked to an example in the narrative so that students can refer to the example in question while solving exercises of a similar nature. The decision to do this was very deliberate: *We want students to read the text*. Our hope is that this decision will increase their chances of understanding the material.
- The static figures, tables, key concepts and definitions found in the text are available within MyMathLab® as PowerPoint® slides. Use them in conjunction with your own prepared slides or as a supplement to what you present at the chalkboard. The authors went to great lengths to provide figures that “speak” to students in order to facilitate geometric intuition and a deeper understanding of calculus. We feel students will benefit from viewing professionally rendered figures in those instances where even the best chalkboard artist among us falls short of the mark. Our advice: invest some time in becoming familiar with these resources, and integrate them into your lectures.
- The interactive figures in the electronic version of the text are also available within MyMathLab. These figures bring alive the concepts of calculus, generate exciting classroom discussions, and provide students with laboratories for further exploration. In short, don’t miss out on them; they will revolutionize the way we teach and the way students learn calculus.

Features of this Guide

The first third of this guide consists of fourteen chapters that correspond to the chapters in the textbook. Each chapter begins with a brief overview of the material covered in the corresponding chapter of the text, sometimes sprinkled with our reasoning for structuring the text as we have. Following this introduction, we provide teaching strategies and classroom activities for each section of the text, as detailed below.

Overview

A quick summary of the section’s content is given to get your bearings.

Lecture Support Notes

Lecture Support Notes are teaching tips linked to each section of the text. We suggest strategies for covering the contents of the section, clarify technical points and terminology, recommend key examples and figures, and on occasion, provide the logic behind the choices we made when writing the book. Everything in this guide is as advertised: a *guide* to teaching your course. It is inevitable that the advice we give is colored by our own preferences and teaching styles. We encourage you to use this guide to complement—but not replace—your unique style of teaching.

It is also important to recognize that the *Lecture Support Notes* were written with the assumption that each section of the book will be covered in full. We acknowledge that this is an unrealistic expectation (see *Optional Sections*, p. vi); our aim is to provide guidance for every section without passing judgment on the relative importance of a particular section’s contents within the calculus curriculum.

Interactive Figures

In this section we list and briefly describe the interactive figures of the electronic book, created by Eric Schulz of Walla Walla Community College, that correspond to the text section at hand. The intent is to let you know what figures are available and to encourage you to integrate them into your classroom presentations. All the interactive figures are accessible through MyMathLab.

Connections

Teaching calculus is a daunting task, especially in the early stages of an instructor's career. It's easy to fall into the trap of compartmentalizing the knowledge you are communicating into disjoint pieces, viewing each section as a separate entity. With *Connections*, we attempt to point out some of the many threads that tie the ideas of calculus together. This component is aimed primarily at instructors (for planning purposes and to see the big picture), but it is also intended for your students. For example, in Section 6.3 of this guide (p. 125) we point out that the general slicing method is used in Chapter 13 to explain the inner workings of an iterated integral and that solids of revolution and their bounding surfaces are featured prominently in multivariable calculus. The first of these facts informs instructors that teaching the general slicing method now will ease the job of explaining iterated integrals in Chapter 13. The second observation may be something you'd like to share with your students. Calling attention to the links between various ideas in calculus will help students view calculus as a unified whole rather than a disparate collection of mathematical facts.

Additional Activities

The entries found under the heading of *Additional Activities* range from five-minute hands-on experiments to detailed guided projects (see *Guided Projects*, pp. iv, viii). Each activity is linked to a particular section, though some activities can be applied to other sections to suit your preferences. A handful of activities that require the use of technology were written with the expectation that *Excel* will be used. We made this choice not out of preference for *Excel* over other available applications (e.g. *Mathematica*, *Maple*, or graphing calculators), but rather because students are likely to have access to *Excel*. These activities are offered for instructors who want a hassle-free means of exposing their students to a taste of technology.

Quick Quizzes

A *Quick Quiz* appears at the end of each section. These quizzes have been carefully written to test the basic facts of the section and are ideal as handouts for your students (in which case you can provide them with answers for self-assessment), in-class quizzes, or Active Learning Questions.

At the conclusion of each chapter, you will find additional support material intended for both students and instructors:

- **Key Terms and Concepts** lists all the major ideas and theorems encountered in the chapter, with page references included. Use this list to construct an exam review sheet, or photocopy it for your students as a quick chapter summary.
- **Review Questions** are designed to probe your students' understanding of the concepts introduced in the chapter rather than test problem solving skills. Though some are certainly appropriate as exam questions, you may find them most useful to generate classroom discussion during an exam review session or as a handout for your students to help them prepare for an exam.
- **Test Bank Exercises** provide material for exams: Cut and paste them into your exam documents or use them as inspiration for your own questions. They can also be used as review material for your students. Understand that the test bank exercises are not meant to be sample exams.

Optional Sections

As noted previously, this guide was written with the assumption that every section of the text will be covered, which is unrealistic given the time constraints in most calculus courses. Following is a list of sections that can be excluded from your syllabus or covered quickly without disrupting the flow of your course. Note that we are not advocating the exclusion of any of this material, nor passing judgment on the importance of these topics in the calculus curriculum—only you can make those decisions for your course. Rather, we are simply identifying those sections that are (largely) unnecessary for moving forward with new material.

- *Chapter 1* This chapter covers prerequisite material for a calculus course, and it can be skipped in its entirety if you'd like to get down to the business of teaching calculus immediately. Our experience is that most students benefit from a review of the algebra and trigonometry skills necessary to survive a calculus course. Therefore, if you choose to skip Chapter 1, it's a good idea to encourage your students to read it and work through its exercises in a self-directed study. Consider supplying a list of exercises that you feel are most important for your course. Sections 3.8 (Derivatives of Inverse Trigonometric Functions) and 7.3 (Trigonometric Substitutions) rely on an understanding of the inverse trigonometric functions. In the event that you omit Chapter 1, it may be wise to cover the second half of Section 1.4 prior to Section 3.8.
- *Section 2.7—Precise Definition of Limits* The remainder of the text does not rely upon an understanding of the ε - δ definition of a limit, and this section may be omitted. That said, if you intend to teach the formal definition of the limit of a sequence (Section 8.2) or limits for multivariable functions (Section 12.3), it's wise to devote some class time to the precise definition of a limit.
- *Section 3.5—Derivatives as Rates of Change* This section fleshes out ideas that are present in other portions of the book, so it can be covered selectively or omitted in its entirety if need be.
- *Section 3.10—Related Rates* As noted in Section 3.10 of this guide (p. 67, *Additional Activities*), most related rates problems can be solved without resorting to the technique normally taught in the classroom. The topic of related rates does not appear again in the text, so this section may be omitted.
- *Section 4.3—Graphing Functions* This section assimilates ideas already presented earlier in the text (in particular, Sections 4.1 and 4.2, but also Sections 1.1, 2.4, 2.5 and 3.1). If you want to teach students how to sketch the graph of a function without devoting an entire section to the topic, extend the examples in Sections 4.1 and 4.2 and skip this section.
- *Section 4.6—The Mean Value Theorem* Though the Mean Value Theorem is a fundamental building block in the theoretical framework of calculus, it can be covered quickly by focusing on what the theorem asserts, its immediate theoretical consequences, and its applicability to problems in the real world. Example 2 and Theorems 4.9 and 4.11 are sufficient for these purposes.
- *Chapter 6* This chapter is devoted to applications of integration, an important component in the calculus curriculum. However, most instructors do not cover all the bases due to time considerations. Rather than listing those topics that may be omitted, here we point out material that is essential for future work. An understanding of how to compute the area of a general region in the plane (Section 6.2) is needed for multiple integrals; we also appeal to the general slicing method from Section 6.3 to explain the mechanics of iterated integrals. Arc length (Section 6.5) should be covered so that arc length parameterizations and line integrals can be understood in Chapter 14. In Section 6.6, mass as the integral of a density function and the concept of work are important ideas for multivariable calculus. The remainder of the material in this chapter can be incorporated into your course as you wish.
- *Section 7.5—Other Integration Strategies* None of the material in this section is required for future work.
- *Section 7.6—Numerical Integration* This section may also be omitted. If you want to be sure your students hear the message that many integrals require numerical methods, recognize that Section 9.4 provides another opportunity to approximate the value of a definite integral by employing power series solutions for integrals such as $\int_0^1 e^{-x^2} dx$.
- *Section 7.8—Introduction to Differential Equations* As long as you avoid exercises in future chapters that include a differential equation component (they are few in number), this section may be omitted.
- *Section 9.4—Working with Taylor Series* This section provides a potpourri of applications of power series, none of which appears again later in the text.
- *Section 10.4—Conic Sections* Most students have encountered conic sections in prior courses. If that describes the population you work with, this section may be quickly reviewed (an understanding of conic sections in Cartesian coordinates is necessary for working with quadric surfaces in Section 12.1).
- *Section 11.7—Motion in Space* Vector-valued functions are used to describe motion in space, and the relationships between position, velocity, and acceleration are useful for interpreting vector fields in Chapter 14. However, much of the remaining material in this section may be treated quickly or omitted.
- *Section 11.9—Curvature and Normal Vectors* The concept of arc length as a parameter is important for future work. The remaining material in this section does not appear again in the text.

- *Section 12.3—Limits and Continuity* A brief discussion of the concepts of limits and continuity for multivariable functions is sufficient for future work.
- *Section 12.9—Lagrange Multipliers* Lagrange multipliers provide another approach to solving optimization problems, but they do not appear again in the text.
- *Section 13.7—Change of Variables in Multiple Integrals* None of this material is required for future work.
- *Chapter 14* Our experience is that reaching the end of Chapter 14 before the term is over requires a minor miracle. The topics in Sections 14.1–14.4 are rather sequential, and therefore it is necessary to cover just about everything in these sections to make sense of Green’s Theorem in both of its forms. Once beyond Green’s Theorem, make an assessment of how much additional information you can fit into the end of your course, and plan accordingly. Strategies for trimming material from the final three sections are discussed in Section 14.6 of this guide (p. 321). If by Section 14.5 it is already apparent you will not make it to the end of the text, consider covering only one of the three-dimensional versions of the divergence and curl, whichever suits your purposes best for your end game.

Guided Projects

The *Guided Projects* section of this guide is a collection of 78 projects that cover a wide range of applications, calculations, and theoretical topics. They are designed to be worked on independently by students or small groups of students in a step-by-step fashion. The projects allow students to step outside the bounds of a typical calculus course and explore related topics. They also provide instructors with an excellent alternative form of assessment.

Answers and Solutions

Answers for all chapter-level content and solutions for the guided projects occupy the last third of this guide. Here you will find answers to the multiple-choice *Quick Quiz* questions, the *Chapter Review Questions*, and the *Test Bank Exercises* from each chapter. Full solutions are provided for each of the guided projects.

Study Cards

The final pages of this guide contain study cards that accompany the textbook, split into single variable and multivariable cards. Post them on your class website or make copies and distribute them to your students.

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