CHAPTER 1

Introduction

1.1

/\*

Exercise 1.1

Selection of integers with k = N/2

select1 => sorting and selecting

select2 => keeping top k

\*/

#include <iostream>

#include <ctime>

#include <cmath>

#include <vector>

#include <algorithm>

using namespace std;

void sort(vector<int> & vec)

{ // bubble sort ascending

bool sorted = false;

while (!sorted)

{

sorted = true;

for (auto i = 1; i < vec.size(); i++)

{

if (vec[i-1]> vec[i])

{

swap(vec[i],vec[i-1]);

sorted = false;

}

}

}

}

void sortDec(vector<int> & vec)

{ // bubble sort descending

bool sorted = false;

while (!sorted)

{

sorted = true;

for (auto i = 1; i < vec.size(); i++)

{

if (vec[i-1]< vec[i])

{

swap(vec[i],vec[i-1]);

sorted = false;

}

}

}

}

int select1(vector<int> nums)

{

int k = (nums.size()+1)/2;

sort(nums);

return nums[k];

}

int select2(const vector<int> &nums)

{

int k = nums.size()/2;

vector<int> topK(nums.begin(), nums.begin() + k);

sortDec(topK);

for (auto i = k; i < nums.size(); i++)

{

if (nums[i] > topK[k-1])

{

for (auto j = k-2; j >=0 ; j--)

if (nums[i] < topK[j])

{topK[j+1] = nums[i]; break;}

else

topK[j+1] = topK[j];

if (topK[0] < nums[i])

topK[0] = nums[i];

}

}

return topK[k-1];

}

int main()

{

vector<int> nums;

int selected;

time\_t start, end;

srand(time(NULL));

for (auto numInts = 1000; numInts<=10000; numInts+=1000)

// sizes 1,000, 2,000, 3,000, ...10,000

{

nums.resize(numInts);

start = time(NULL);

for (auto i = 0; i < 10; i++) // run 10 times

{

for (auto j = 0; j < numInts; j++)

nums[j] = rand()%(2\*numInts);

selected = select1(nums); // or selected = select2(nums);

}

end = time(NULL);

cout<<numInts<<"\t"<<difftime(end,start)<<endl;

}

return 0;

}

2. /\*

Word Puzzle problem

from the example in figure 1.1

\*/

#include<iostream>

#include<fstream>

#include<string>

#include<vector>

#include "matrix.h"

#include<algorithm>

using namespace std;

const int MAXROWS = 4;

const int MAXCOLS = 4;

struct Orientation

{

Orientation() : delRow(0), delCol(0) {}

Orientation operator() (int direction)

{

switch (direction)

{

case 0 : delRow = -1; delCol = -1; break;

case 1 : delRow = -1; delCol = 0; break;

case 2 : delRow = -1; delCol = 1; break;

case 3 : delRow = 0; delCol = -1; break;

case 4 : delRow = 0; delCol = 1; break;

case 5 : delRow = 1; delCol = -1; break;

case 6 : delRow = 1; delCol = 0; break;

case 7 : delRow = 1; delCol = 1; break;

}

return \*this;

}

int delRow;

int delCol;

};

class Puzzle

{

public:

Puzzle(int numRows, int numCols )

{

matrix<char> temp(numRows,numCols);

puzzle= temp;

initPuzzle();

}

Puzzle(int numRows , int numCols , vector<string> wordList) : dictionary(wordList)

{

matrix<char> temp(numRows,numCols);

puzzle= temp;

initPuzzle();

}

void solvePuzzle();

void findWords(int startRow, int startCol, Orientation orient);

private:

void initPuzzle();

matrix<char> puzzle;

vector<string> dictionary;

};

void Puzzle::initPuzzle()

{

puzzle[0][0] = 't';

puzzle[0][1] = 'h';

puzzle[0][2] = 'i';

puzzle[0][3] = 's';

puzzle[1][0] = 'w';

puzzle[1][1] = 'a';

puzzle[1][2] = 't';

puzzle[1][3] = 's';

puzzle[2][0] = 'o';

puzzle[2][1] = 'a';

puzzle[2][2] = 'h';

puzzle[2][3] = 'g';

puzzle[3][0] = 'f';

puzzle[3][1] = 'g';

puzzle[3][2] = 'd';

puzzle[3][3] = 't';

}

void Puzzle::solvePuzzle()

{

Orientation orient;

for ( auto startRow = 0; startRow < puzzle.numrows(); startRow++)

for ( auto startCol=0; startCol < puzzle.numcols(); startCol++)

for (auto i = 0; i < 8 ; i++)

findWords(startRow,startCol,orient(i));

}

void Puzzle::findWords(int startRow, int startCol, Orientation orient)

{

string word ="";

int row = startRow;

int col = startCol;

do

{

word = word + puzzle[row][col];

if (find(dictionary.begin(), dictionary.end(), word) != dictionary.end())

cout<<word<<" found starting at ("<<startRow<<","<<startCol<<")\n";

row += orient.delRow;

col += orient.delCol;

} while (row > -1 && col > -1 && row < puzzle.numrows() && col < puzzle.numcols());

}

int main()

{

string diction[] = {"this", "two", "fat", "fats", "at", "wad", "ad", "hat", "that", "his","is","it","ah"} ;

vector<string> dictionary(diction,diction+ 12);

Puzzle puzzle(MAXROWS, MAXCOLS, dictionary);

puzzle.solvePuzzle();

return 0;

}

1.3

void printDouble(double x)

{

if (x < 0)

{

cout<<"-";

x = -x;

}

int intPart = floor(x);

double fract = x - intPart;

printOut(intPart);

cout<<".";

while (fract<1 && fract > 0.0000000001)// 0.0000000001 is machine accuracy.

{

fract \*= 10;

printDigit(floor(fract));

fract = fract - floor(fract);

}

}

1.4

The general way to do this is to write a procedure with heading

void processFile( String fileName );

which opens fileName, does whatever processing is needed, and then closes it. If a line of the form

#include SomeFile

is detected, then the call

processFile( SomeFile );

is made recursively. Self-referential includes can be detected by keeping a list of files for which a call to processFile has not yet terminated, and checking this list before making a new call to processFile.

1.5

**int ones( int n )**

**{**

**if( n < 2 )**

**return n;**

**return n % 2 + ones( n / 2 );**

**}**

1.6

void permute(const string & str, int low, int high)

{

char letter;

string tmp = str;

if (low >= high)

cout<<str<<endl;

else

{

for (auto i= low; i < str.size(); i++)

{

swap(tmp[0], tmp[i]);

permute(tmp, low+1, high);

}

}

}

void permute(const string & str)

{

permute(str, 0, str.size());

}

1.7 (a) The proof is by induction. The theorem is clearly true for 0 < X ≤ 1, since it is true for X = 1, and for X < 1, log X is negative. It is also easy to see that the theorem holds for 1 < X ≤ 2, since it is true for X = 2, and for X < 2, log X is at most 1. Suppose the theorem is true for p < X ≤ 2p (where p is a positive integer), and consider any 2*p* < Y ≤ 4*p* (*p* ≥ 1). Then log Y = 1 + log(Y/2)< 1 + Y/2 < Y/2 + Y/2 ≤ Y, where the first inequality follows by the inductive hypothesis.

(b) Let 2*X* = *A*. Then AB = (2X)B = 2XB. Thus log AB = XB. Since X = log *A*, the theorem is proved.

1.8 (a) The sum is 4/3 and follows directly from the formula.

(b)  Subtracting the first equation from the second gives By part (a), 3*S* = 4/3 so S = 4/9.

(c)  Subtracting the first equation from the second gives  Rewriting, we get  Thus 3*S* = 2(4/9) + 4/3 = 20/9. Thus S = 20/27.

(d) Let SN =  Follow the same method as in parts (a) – (c) to obtain a formula for SN in terms of *SN–*1, *SN–*2,..., *S*0 and solve the recurrence. Solving the recurrence is very difficult.

1.9 

1.10 24 = 16  1 (mod 5). (24)25  125 (mod 5). Thus 2100  1 (mod 5).

1.11 (a) Proof is by induction. The statement is clearly true for N = 1 and N = 2. Assume true for N = 1, 2, ... , k. Then  By the induction hypothesis, the value of the sum on the right is *Fk*+2 – 2 + *Fk*+1 = *Fk*+3 – 2, where the latter equality follows from the definition of the Fibonacci numbers. This proves the claim for N = k + 1, and hence for all *N*.

(b) As in the text, the proof is by induction. Observe that *φ* + 1 = *φ*2. This implies that *φ –*1 + *φ –*2 = 1. For N = 1 and N = 2, the statement is true. Assume the claim is true for N = 1, 2, ... , k.



by the definition, and we can use the inductive hypothesis on the right-hand side, obtaining



and proving the theorem.

(c) See any of the advanced math references at the end of the chapter. The derivation involves the use of generating functions.

1.12 (a)  = N(N + 1) – N = *N*2.

(b) The easiest way to prove this is by induction. The case N = 1 is trivial. Otherwise,



**1.15**

class EmployeeLastNameCompare

{

public:

bool operator () (const Employee & lhs, const Employee & rhs) const

{ return getLast(lhs.getName())< getLast(rhs.getName());}

};

string getLast( const string & name)

{

string last;

int blankPosition = name.find(" ");

last = name.substr(blankPosition+1, name.size());

return last;

}

int main()

{

vector<Employee> v(3);

v[0].setValue("George Bush", 400000.00);

v[1].setValue("Bill Gates", 2000000000.00);

v[2].setValue("Dr. Phil", 13000000.00);

cout<<findMax(v, EmployeeLastNameCompare())<<endl;

return 0;

}

**1.16**

matrix() : array(10)

{for( auto & thisRow : array )

thisRow.resize( 10 );

}

void resize(int rows, int cols)

{

array.resize(rows);

for ( auto & thisRow : array)

thisRow.resize(cols);

}